

Noise Kernel for Reissner Nordström Metric: Results at Cauchy Horizon

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Abstract

We obtain point separated Noise Kernel for the Reissner Nordström metric. The Noise Kernel defines the fluctuations of the quantum stress tensor and is of central importance to Semiclassical Stochastic Gravity. The metric is modeled as gravitationally collapsing spacetime, by using suitable coordinate transformations, defined earlier. The fluctuations of the quantum stress tensor, at the final stage of collapse are then analysed for both, the naked singularity and black hole end states. The behavior of this Noise Kernel, at the Cauchy Horizon for naked singularity shows markedly different behaviour from self similar Tolman Bondi metric, which was obtained earlier. In the latter a very unique divergence was seen, which does not appear for the Reissner Nordström metric, here. It is known that the quantum stress tensor itself, diverges at the Cauchy Horizon (CH) for both of these metrics. In contrast, it can now be seen that the the fluctuations of the stress tensor behave differently for the two cases. We give a discussion and further directions for investigations of this interesting behaviour in the two cases (regarding the collapse scenario).

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1 Introduction

The Cosmic Censorship Conjecture (CCC) is considered to be an important open area of research since its inception in 1969. The precise mathematical formulation of this hypothesis is much awaited. The two possible end states for complete gravitational collapse of a massive star, namely black hole (BH) and naked singularity (NS), need an elaborate and varied analysis for the confirmation of the CCC or otherwise. There are several studies in this direction both numerical and analytical, which span classical and quantum domains [1, 2, 3, 4, 5]. What prevents naked singularities to occur in nature, according to the CCC is a quest.

However, it is important to note that, the naked singularities arise not only as solutions to the Einstein's equations for a collapsing star, but are also known to have observational consequences [6, 7].

The genericity and stability criteria for naked vs covered singularities is a vast literature [8, 9, 10, 11] nevertheless, some new insights can shed light on this study from different perspectives.

After the formulation of Semiclassical Stochastic Gravity [12] in late 90's, the current focus now is over the applications of the theory to Cosmology and black hole (BH) physics. The Noise Kernel, defines the fluctuations of the quantum stress tensor in a given spacetime, and is of central importance to Stochastic Gravity [13]. In this mean field approach towards gravity, the background spacetime plays the role of the system, while the quantum states that of the environment.

Stochastic Gravity is very new to the study of CCC, and has recently been seen to contribute towards interesting directions in this regard. The first of its applications to CCC has been carried out in [14], for the spherically symmetric dust (self-similar Tolman Bondi metric). The results therein show a marked difference in the behaviour of fluctuations of the quantum stress tensor, for the two possible end states. One sees a unique divergence at the CH for the Naked Singularity (NS) while no such divergence is seen if the collapse results in a black hole. This result awaits explanation for such a behaviour at CH, where the validity of stochastic semiclassical gravity breaks down.

In this article, we attempt to find out if a similar divergence at the CH, occurs in case of Reissner Nordström metric also. This would further help investigations towards the earlier obtained result and indicate if such a divergence is a generic feature of naked singularities.

One would expect such a divergence of the Noise Kernel at the CH for all cases, should that be a peculiar behaviour for all NS end states. Incidentally, we obtain a very different result in the case of Reissner Nordström metric. No such divergence of the Noise Kernel is seen here. This raises a lot of further interest, into underlying reasons for such behaviour. Different spacetimes metrics can be associated with complete gravitational collapse that admit NS and BH solutions as end states.

This study gives directions for more to be explored, and a theme which looks into specific physical conditions responsible for the kind of difference that we see. Possible reasons for the difference in the behavior of fluctuations of stress tensor at the CH, for the two spacetimes (namely Tolman Bondi and RN metric) are given towards the end of the article.

The article is organised as follows.

In section (2.1) we present the Reissner Nordström metric solution in a form, more suitable for the collapse scenario. In section (2.2) a short description of the Noise Kernel, defining the fluctuations of stress tensor is given. In the subsequent section (2.3) the relevant quantum states associated with the stress tensor and related issues are addressed. The metric is obtained in appropriate form, which is convenient for the calculation technique used for Noise Kernel. Section (3) gives the physical interpretation of the result obtained at the CH. In the last section, we give the conclusions that can be drawn from the result obtained and discuss some important issues.

2 Evaluating the Noise Kernel for Reissner Nordström metric

In the following, we attempt to calculate the Noise Kernel components for the Reissner Nordström metric in detail. The procedure for this and relevant issues are reviewed. As mentioned later, a symbolic code has been used to evaluate the final expressions.

2.1 The Reissner Nordström metric and gravitational collapse

The Reissner Nordström (RN) solution in coordinates suitable for dynamical evolution of the initial data is developed in [15]. This is necessary for the gravitational collapse scenario applied to the spacetime metric. We briefly review these coordinates and the metric structure in the following.

The Reissner Nordström solution can be treated as the evolution of a spherically symmetric metric with charge E and mass M from a regular initial data for $E < M$. This metric can be written in coordinates which mimic dynamical evolution. The background metric, along with suitable quantum states, describes the semiclassical behaviour of such an evolution. This can be viewed as resulting from initial data, evolving as a collapse leading to curvature singularity.

The RN metric is usually represented as

$$ds^2 = P(r)dt^2 - P(r)^{-1}dr^2 - r^2d\Omega^2 \quad (1)$$

where $P(r) = 1 - 2M/r + E^2/r^2$. As introduced in [15] the special coordinates defined by

$$T = t + \int \frac{g(r)}{P(r)} dr \quad (2)$$

$$R = t + \int \frac{1}{g(r)P(r)} dr \quad (3)$$

are useful in casting the metric as initial value problem. The function $g(r)$ has been introduced and engineered so as to ensure regularity of the transformations proposed in the above reference. It is chosen in such a way so that $P(r)/(1 - g^2(r)) > 0$, this maintains the signature of the transformed metric.

The metric thus takes the form.

$$ds^2 = \frac{P(r)}{1 - g(r)^2} dT^2 - \frac{g(r)^2 P(r)}{1 - g(r)^2} dR^2 - r^2 d\Omega^2 \quad (4)$$

Since we aim at analysing the same physical situation of the gravitational collapse as in [15], we intend to use the same coordinates.

Further we use the double null coordinates ,

$$u = T - g(r)R, v = T + g(r)R \quad (5)$$

to obtain the form

$$ds^2 = \frac{P(r)}{1 - g(r)^2} dudv + r^2 d\Omega \quad (6)$$

for the metric.

These coordinates have been used to analyse the behavior of quantum stress tensor near the cauchy horizon. It is seen in the above reference that the stress tensor diverges at the CH. It remains regular at the event horizon when a covered singularity is formed. Though the CH is unstable here, the study of fluctuations is of interest because it probes the extended spacetime structure in the neighbourhood of the CH, as explained later. The fluctuations of the stress tensor, which are responsible for the induced fluctuations of the metric, can be characterised by the Noise Kernel in the Einstein Langevin formalism.

(A similar study for the Tolman Bondi metric shows that the quantum stress tensor and noise kernel [16, 14] both diverge at the CH.)

For regions near the CH, $g = g_1$ (details as in [15]) in the above equation and so we can write

$$ds^2 = \frac{P(r)}{1 - g_1^2} dudv + r^2 d\Omega \quad (7)$$

Next, we give a brief description of the Noise Kernel bitensor, which forms a quantity of central interest in our work and Stochastic gravity.

2.2 The Noise Kernel

The Einstein Langevin Equation defining Semiclassical stochastic gravity, is an extension to the theory of semiclassical gravity. The inclusion of fluctuations of the quantum stress tensor, gives semiclassical theory a Langevin approach. These induced fluctuations to the metric, play an important role in determining extended structure of the spacetime. The Einstein Langevin equation reads

$$G_{ab} + \Lambda g_{ab} = \langle \hat{T}_{ab} \rangle + \hat{\xi}_{ab} \quad (8)$$

(we use the convention $c = G = \hbar = k_B = 1$) where expectation is taken over a normalized state and $\hat{\xi}_{ab}$ is a random variable. This is characterised by

$$\langle \hat{\xi}_{ab}(x) \rangle_s = 0, \langle \hat{\xi}_{ab}(x) \hat{\xi}_{cd}(x') \rangle_s = N_{abc'd'}(x, x') \quad (9)$$

The source in the above, thus is gaussian and the bitensor $N_{abc'd'}(x, x')$ expression can be obtained explicitly as follows

$$N_{abc'd'}(x, x') = \frac{1}{2} \langle \{ \hat{t}_{ab}(x), \hat{t}_{c'd'}(x') \} \rangle \quad (10)$$

where

$$\hat{t}_{ab}(x) \equiv \hat{T}_{ab}(x) - \langle \hat{T}_{ab}(x) \rangle$$

The general expressions for the Noise Kernel in non-coincident limit have been obtained in [13], giving various cases of couplings, for massive as well as massless fields.

Here we are interested in calculating the Noise Kernel for conformally invariant scalar field.

To begin with, the classical stress tensor of a conformally invariant scalar field ϕ is given by

$$T_{ab} = \nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} \nabla^c \phi \nabla_c \phi + \frac{1}{6} (g_{ab} \square - \nabla_a \nabla_b + G_{ab}) \phi^2 \quad (11)$$

The scalar field after being quantised is treated as an operator, while the metric g_{ab} is treated classically.

The quantum states need to be specified for conformally invariant fields. The Noise Kernel can then be obtained in terms of Wightman function [13], and the expression is given as

$$N_{abc'd'} = Re\{\bar{K}_{abc'd'} + g_{ab} \bar{K}_{c'd'} + g_{c'd'} \bar{K}'_{ab} + g_{ab} g_{c'd'} \bar{K}\} \quad (12)$$

$$\begin{aligned} 9\bar{K}_{abc'd'} = & 4(\mathcal{G}_{;c'b} \mathcal{G}_{;d'a} + \mathcal{G}_{c'a} \mathcal{G}_{;d'b}) + \mathcal{G}_{;c'd'} \mathcal{G}_{;ab} + \mathcal{G} \mathcal{G}_{;abc'd'} - \\ & 2(\mathcal{G}_{;b} \mathcal{G}_{c'ad'} + \mathcal{G}_{;a} \mathcal{G}_{c'bd'} + \mathcal{G}_{;d'} \mathcal{G}_{;abc'} + \mathcal{G}_{;c'} \mathcal{G}_{;abd'}) + 2(\mathcal{G}_{;a} \mathcal{G}_{;b} R_{c'd'} + \\ & \mathcal{G}_{;c'} \mathcal{G}_{;d'} R_{ab}) - (\mathcal{G}_{;ab} R_{c'd'} + \mathcal{G}_{;c'd'} R_{ab}) \mathcal{G} + \frac{1}{2} R_{c'd'} R_{ab} G^2 \end{aligned} \quad (13)$$

$$\begin{aligned} 36\bar{K}'_{ab} = & 8(-\mathcal{G}_{;p'b} \mathcal{G}'_{;a} + \mathcal{G}_{;b} \mathcal{G}'_{;p'a} + \mathcal{G}_{;a} \mathcal{G}'_{;p'b}) + \\ & 4(\mathcal{G}'_{;p'} \mathcal{G}_{;abp'} - \mathcal{G}'_{;p'} \mathcal{G}_{;ab} - \mathcal{G} \mathcal{G}'_{;abp'}) - 2R'(2\mathcal{G}_{;a} \mathcal{G}_{;b} - \mathcal{G} \mathcal{G}_{;ab}) \\ & - 2(\mathcal{G}_{;p'} \mathcal{G}_{;p'} - 2\mathcal{G} \mathcal{G}'_{;p'}) R_{ab} - R' R_{ab} G^2 \end{aligned} \quad (14)$$

$$\begin{aligned} 36\bar{K} = & 2\mathcal{G}_{;p'q} \mathcal{G}'_{;q} + 4(\mathcal{G}'_{;p'} \mathcal{G}_{;q} + \mathcal{G} \mathcal{G}'_{;pq'}) - 4(\mathcal{G}_{;p} \mathcal{G}'_{;q'} + \mathcal{G}'_{;q} \mathcal{G}_{;qp'}) \\ & + R \mathcal{G}_{;p'} \mathcal{G}'_{;p'} + R' \mathcal{G}_{;p} \mathcal{G}'_{;p} - 2(R \mathcal{G}'_{;p'} + R' \mathcal{G}_{;p}) \mathcal{G} + \frac{1}{2} R R' G^2 \end{aligned} \quad (15)$$

Here the Noise Kernel obeys the following properties [17].

1. $N_{abc'd'}(x, x') = N_{c'd'ab}(x', x)$
2. $\nabla^a N_{abc'd'} = \nabla^b N_{abc'd'} = \nabla^{c'} N_{abc'd'} = \nabla^{d'} N_{abc'd'} = 0$.
3. $N^a_{ac'd'} = N_{ab}{}^{c'}{}_{c'} = 0$.
4. The Noise Kernel is semidefinite ,

$$\int d^4x \sqrt{-g(x)} \int d^4x' \sqrt{-g(x')} f^{ab}(x) N_{abc'd'}(x, x') f^{c'd'}(x'x) \geq 0$$
for any real tensor field $f^{ab}(x)$.

2.3 The Quantum states and Wightman functions

The quantum states play a decisive role in the behavior of the fluctuations of the matter fields . In our study, we try to use the most general class of quantum states for non-interacting fields. These fulfill the basic requirements for a well defined stress tensor and its fluctuations, in the curved spacetime background.

We therefore use, the Quasifree thermal states of Hadamard type in this work. The states being Hadamard ensures that the stress tensor is well defined within maximal Cauchy development and obeys Wald axioms.

The Wightman two point function in (13) - (15) is given by

$$\mathcal{G}(x, x') = \langle \phi(x)\phi(x') \rangle \quad (16)$$

this determines the quantum state of the field if we work with quasi-free (Gaussian) states.

The state being thermal or KMS type assigns a temperature κ . This leads to Wightman functions, being approximated for thermal states via the Gaussian approximation (Page's approximation) as described in [17]. The temperature is given by

$$T = \frac{\kappa}{2\pi} \quad (17)$$

An appropriate form for Wightman function for given spacetime is required to obtain the noise kernel. In general, numerical methods can be used to get this for arbitrary separations. For small separations however, analytical form for the expressions can be obtained by using an approximate method. This has been established in earlier work, and we intend to use Page's approximation as has been done for other spacetimes [14, 17].

The Wightman function for ultrastatic spacetime, which can be related conformally to the Reissner Nordström metric is calculated. This is then used to obtain the Noise Kernel for the same. These Wightman functions are, as shown below, expressed in terms of the synge function. However, first we need to put the metric in the desired form as would be used.

For this, we transform the Reissner Norstdrom metric to ultrastatic form. Following equation (7)

$$ds^2 = P(r)du_+dv + r^2d\Omega \quad (18)$$

where $du_+ = du/(1 - g_1^2)$,

let $P(r)/r^2du_+ = dU$, then

$$ds^2 = r^2[dUdv + d\Omega] \quad (19)$$

Further transforming as,

$$T = U - v \text{ and } X = U + v ,$$

$$U = \frac{T + X}{2}, v = \frac{X - T}{2}$$

which gives

$$dUdv = \frac{1}{4}(-dT^2 + dX^2)$$

the form of the metric is now

$$ds^2 = r^2\left\{-\frac{1}{4}dT^2 + \frac{1}{4}dX^2 + d\Omega\right\} \quad (20)$$

This is in conformal ultrastatic form,

$$ds^2 = \Sigma^2 \left\{ -\frac{1}{4}dT^2 + \frac{1}{4}dX^2 + d\Omega \right\} \quad (21)$$

where the conformal factor $\Sigma = r^2$. We compare this with the ultrastatic metric which takes the form

$$ds^2 = dt^2 + g_{ij}(\vec{x})d\vec{x}^i d\vec{x}^j \quad (22)$$

where metric functions g_{ij} are independent of time t . For this metric the synge function takes the form

$$\sigma(x, x') = \frac{1}{2}((t - t')^2 - \mathbf{r}^2) \quad (23)$$

where \mathbf{r}^2 is twice the spatial part of the Synge function, which depends only on the spatial coordinates.

The expression for Wightman function in ultrastatic background metric for a KMS state can be calculated under Gaussian approximation [17]

$$\mathcal{G}(\Delta t, x, x') = \frac{\kappa \sinh(\kappa \mathbf{r})}{8\pi \mathbf{r} [\cosh(\kappa \mathbf{r}) - \cosh(\kappa \Delta t)]} U(\Delta t, x, x') \quad (24)$$

This can be expanded as

$$\mathcal{G}(x, x') = \frac{1}{8\pi^2} \left[\frac{1}{\sigma} + \frac{\kappa^2}{6} - \frac{\kappa^4}{180} (2(\Delta t)^2 + \sigma) + O[(x - x')^4] \right] U(x, x') \quad (25)$$

where

$$\begin{aligned} U(x, x') &= \Delta^{1/2}(x, x') \\ \Delta(x, x') &= \frac{1}{\sqrt{-g(x)}\sqrt{-g(x')}} \det(\sigma_{;ab'}) \end{aligned} \quad (26)$$

The expressions as presented above, are made up of the synge function. For the above metric (20) in ultrastatic form, we calculate this synge function and it takes the form

$$\sigma = -\frac{1}{2}(T' - T)^2 + (X' - X)^2 + \eta^2 \quad (27)$$

where

$$\cos(\eta) = \cos(\theta) \cos(\theta') + \sin(\theta) \sin(\theta') \cos(\phi - \phi')$$

Given this function as in (27), we substitute it in equation (26) and get

$$U(x, x') = \sqrt{\frac{\eta}{\sin(\eta)}} \quad (28)$$

The Noise Kernel components for the RN metric can now be computed using the expressions above.

2.4 Noise Kernel for Reissner Nordström metric

Substituting equation (27) and (28) in (25), and finally the expression for Wightman function thus obtained in (13) (14), (15) and (12) different components of the Noise Kernel can be obtained.

We display here, one of the components thus evaluated. Other components are similar in nature, as far as the results at the CH discussed in this paper are concerned. The following component $N_{TT'T'}$ which we show below, is point separated with $\eta = 0$ and $\delta X = 0$, while $\delta T \neq 0$. The result below is obtained after conformal transformation of Noise Kernel from ultrastatic to the original spacetime,

$$N_{abcd}(x, x') = \Sigma^{-2}(x) \tilde{N}_{abcd}(x, x') \Sigma^{-2}(x') \quad (29)$$

$$\begin{aligned} N_{TT'T'} = & \frac{1}{r^2(T, X)r^2(T', X)} [\kappa^0 \left(\frac{2897}{288\pi^2\delta T^8} - \frac{1}{72\pi^2\delta T^7} + \frac{349}{1152\pi^2\delta T^6} - \frac{7}{3456\pi^2\delta T^5} \right. \\ & \left. + \frac{116640000 + 13934592000\pi}{15925248000\pi^2\delta T^4} \right) \\ & \kappa^2 \left(\frac{1}{96\pi^2\delta T^8} + \frac{211}{864\pi^2\delta T^6} - \frac{1}{864\pi^2\delta T^5} + \frac{7}{512\pi^2\delta T^4} \right) \\ & \kappa^4 (1/(1152\pi^2\delta T^6) + 1133/(34560\pi^2\delta T^4)) \\ & \left. + \kappa^6 (1/(34560\pi^2\delta T^4)) \right] \end{aligned} \quad (30)$$

These expressions have been evaluated using symbolic code developed for the same. One can check correctness of the result by using properties of the Noise Kernel given in the earlier section.

3 Interpretation of the Noise Kernel expression at the Cauchy Horizon

The equation for the Cauchy Horizon is given by $P(r) = 0$, where the smaller root $r = r_-$ as shown in the figure defines the first null ray (i.e the CH) coming out of the singularity [18]. If we take $r(T, X) = r_-$, placing one point on the CH and the other at $r(T', X)$ separated from it (as valid under gaussian approximation) we see that the Noise Kernel is regular everywhere including the CH. This result is markedly different from that of the Tolman Bondi metric. We ensure that all components of the point separated Noise Kernel for Reissner Nordström metric are regular at the CH and no divergences are seen to occur.

The importance of the above result lies in realising that, though the Cauchy Horizon here is unstable, we can draw some very important conclusions on the regions of spacetime very near to the naked singularity. Any other analysis fails or is invalid in this region. The noise kernel is the correlation of the fluctuations at two different points. The above calculation is valid only for spacelike or time like separations. This ensures that only one point can be put on the CH, and the other should be placed on a timelike or a spacelike separation nearby. We see that this correlation is finite, even though the CH itself is

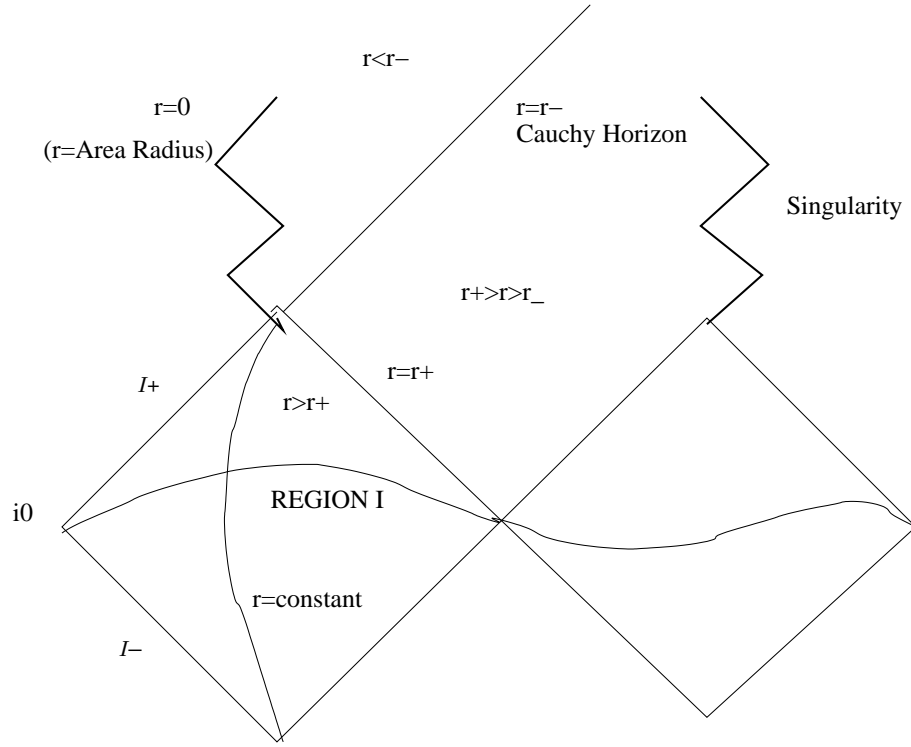


Figure 1: Penrose diagram of the Reissner Nordström Metric for the case $E < M$. Cauchy surface across region I is shown by the curved line. The CH is shown by $r = r_-$.

unstable with the quantum stress tensor also being divergent. The result thus obtained indicates that, using the Einstein Langevin approach it is still possible to probe the extended structure of spacetime in terms of backreaction studies. More importantly, this is possible at the stochastic semiclassical level, where quantum gravity effects set in. (This is certainly not possible for the Tolman Bondi metric, since the noise kernel is divergent there).

On the other hand, noise kernel divergence (as for Tolman Bondi metric) indicates large contributions of fluctuations near CH. The question which would naturally arise is regarding the nature of probability distribution of the stress tensor (which is treated as a random variable). In 3+1-dimensions, a closed form expression for the probability distribution for the stress tensor is not available, but a certain amount is known about the behaviour of the moments of the energy density e.g., for massless scalar and electromagnetic fields. This implies that the probability distribution has a slowly decaying tail at large positive values [19]. Ford argued from this, that large positive fluctuations would dominate over thermal fluctuations at large energies, with potential implications for rate estimates of black hole nucleation from the vacuum.

The noise kernel for a thermal state becoming very large would imply that thermal fluctuations seem to dominate near CH. This result is complementary to Ford's argument above.

The question that one could ask here is, whether the noise kernel blows up at every Cauchy horizon. This paper shows that the answer is negative. The Reissner Nordström metric is an exception.

4 Conclusion and further Directions

The CH of the RN metric is important in some other ways as well. The classical phenomenon of mass inflation of the RN singularity has been well known. The large blue shift of infalling radiation leads to such a phenomenon. Physically, particle creation must also be addressed, especially when the blue shift energies are very high. Quantum stress tensor is a way to address both blue shift energy as well as particle creation effects together. But the quantum stress tensor also diverges [15] upholding the physical implication that the CH will behave like a singular light like surface (what Penrose terms as a "thunderbolt"). In this analysis, quantum fluctuations had not been addressed. Like dust collapse where divergent contributions of fluctuations put in doubt the average quantum stress results, we are compelled to ask if fluctuations call into question the idea of thunderbolts or mass inflation. Our results show that they do not.

We have obtained the expressions for Noise Kernel in case of Reissner Nordström metric while it is been modeled for complete gravitational collapse. The Noise Kernel is regular everywhere in the spacetime and thus backreaction studies, using the Einstein Langevin equation can be carried out here. It would be interesting to see the backreaction effects of induced fluctuations of the stress tensor at the Cauchy Horizon. This is a very important result, for various reasons.

- The quantum stress tensor for the RN metric diverges (ref. to earlier work) at the CH, while the fluctuations of the same do not. This indicates that the blowing up of the quantum stress tensor can be interpreted as energy burst more appropriately in the case of RN metric as has been suggested. While for the Tolman Bondi metric the divergence of the stress tensor as well as its fluctuations do not give clear indication for the energy burst at the CH.
- One may observe that, quantum fluctuations diverge at CH, in the case where classical stress tensor is non zero. While it is not so when the classical stress tensor is zero. In order to confirm this behaviour, we suggest few more examples of spacetimes to be worked out on similar lines. Further investigations can be carried out based on the results thus obtained. We intend to explore in this direction in future.
- It is important to note that this distinction in the gravitational collapse for self similar Tolman Bondi and Reissner Nordström metric is seen only for the "fluctuations" of the quantum stress tensor, while the quantum stress tensor itself is divergent in both cases (as obtained in earlier work).
- Corresponding to the collapse scenario, difference between the two metrics compared here, occurs first at the classical level, while remaining same at the semiclassical level and then reappearing at the stochastic semiclassical level. It is important to ponder over the reason and mechanism, how the classical and the quantum matter fields are connected together to the spacetime geometry and if they influence each other via the background spacetime. This may lead one to investigate the basic structure of spacetime geometry and related matter fields in the non-linear Einstein equation at the classical , semiclassical and stochastic level. Based on this it may further be interesting to address fundamental issues relating these three domains.

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